AIEEE 2008 Maths Solutions

1 Let p be the statement "x is an irrational number" , q be the statement "y is a transcendental number" , and r be the statement "x is a rational number if y isa transcendental number".

Statement-*1*:

R is equivalent to either q or p .

Statement *2*:

R is equivalent to $\sim(p \longleftrightarrow \sim q)$

(1) Statement -1 is true , Statement -2 is false

(2) Statement -1 is false, Statement -2 is true

(3) Statement -1 is true, Statement -2 is true: Statement -2 is a correct explanation for

Statement -1

(4) Statement -1 is true, Statement -2 is true: Statement -2 is ***not*** a correct explanation for Statement -1

2 In a shop there are five types of ice-creams available. A child buys six ice-creams.

Statement -1 :

The number of different ways the child can buy the six icecreams is $^{10}C_5$.

Statement -2 :

The number of different ways the child can buy the six ice-creams is equal of different ways of arranging 6 A's and 4 B's in a row

(1) Statement -1 is true, Statement -2 is false

(2) Statement -1 is false, Statement -2 is true

(3) Statement -1 is true , Statement -2 is true

Statement -2 is a correct explanation for Statement -1

(4) Statement -1 is true , Statement-2 is true :

Statement -2 is ***not*** a correct explanation for Statement –1

Ans

x_1, x_2, x_3, x_4, x_5 be the number of icecreams selected from 5 types of icecreams

$x_1 + x_2 + x_3 + x_4 + x_5 = 6$

$x_i \geq 0$, I = 0, 1, 2, 3, 4, 5, 6

solve to get number of ways = ${}^{6+5-1}C_{5-1} = {}^{10}C_4$

=> statement (1) is false but statement (2) is correct

3. *Statement-1 :*

$$\sum_{r=0}^{n} (r+1)\,{}^{n}C_r = (n+2)\,2^{n-1}.$$

Statement-2 :

$$\sum_{r=0}^{n} (r+1)\,{}^{n}C_r\, x^r = (1+x)^n + nx\,(1+x)^{n-1}.$$

(1) Statement-1 is true, Statement-2 is false

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

Ans

$$\sum_{r=0}^{n}(r+1)^{n}C_r = \sum_{r=0}^{n} r\,{}^{n}C_r + \sum_{r=0}^{n}{}^{n}C_r$$

$= n2^{n-1} + 2^n = 2^{n-1}(n+2)$, so statement 1 is correct

$$(1+x)^n = \sum_{r=0}^{n}{}^{n}C_r x^r$$

Multiply with x & differentiate

$$\frac{d}{dx}x(1+x)^n = \sum_{r=0}^{n}(r+1)^{n}C_r x^r$$

$$(1+x)^n + nx(1+x)^{n-1} = \sum_{r=0}^{n} (r+1)^n C_r x^r$$

statement 2 is correct

4. *Statement-1 :*

For every natural number $n \geq 2$,

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{n}} > \sqrt{n}.$$

Statement-2 :

For every natural number $n \geq 2$,

$$\sqrt{n(n+1)} < n + 1.$$

(1) Statement-1 is true, Statement-2 is false

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

Ans

Statement-2: $\sqrt{n(n+1)} < (n+1) \Rightarrow \sqrt{n}\sqrt{n+1} < \left(\sqrt{n+1}\right)^2$

$\sqrt{n} < \sqrt{n+1}$ Always true

Statement (2) is correct

$$\sqrt{n} < \sqrt{n+1} \Rightarrow \frac{1}{\sqrt{n}} > \frac{1}{\sqrt{n+1}}$$

$$\Rightarrow \frac{1}{\sqrt{1}} > \frac{1}{\sqrt{2}} > \frac{1}{\sqrt{3}} --- > \frac{1}{\sqrt{n}} --- (1)$$

statement-1:

$$LHS = \frac{1}{\sqrt{1}} > \frac{1}{\sqrt{2}} > \frac{1}{\sqrt{3}} --- > \frac{1}{\sqrt{n}}$$

$$LHS > \frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n}} --- \frac{1}{\sqrt{n}}$$ using (1)

$$\Rightarrow LHS > \frac{n}{\sqrt{n}} \Rightarrow LHS > \sqrt{n}$$

statement 1 is correct

statement 1 is correct using statement 2

5. Let A be a 2 × 2 matrix with real entries. Let I be the 2 × 2 identity matrix. Denote by tr(A), the sum of diagonal entries of A. Assume that $A^2 = I$.

Statement-1 :
If $A \neq I$ and $A \neq -I$, then $\det A = -1$.

Statement-2 :
If $A \neq I$ and $A \neq -I$, then $tr(A) \neq 0$.

(1) Statement-1 is true, Statement-2 is false

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

Ans

Let

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = A$$

$A^2 = I$

=>

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

=>

$$\begin{bmatrix} a^2+bc & ab+bd \\ ac+dc & bc+d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

=>c[a+d] = 0, b[a+d] = 0, d^2+bc = 1, a^2+bc=1

As $b \neq 0, c \neq 0$, a+d=0 => a = -d

$$\text{Det}(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

=ad –bc = -d^2 –bc = -(d^2 + bc) = -1(as d^2 + bc = -1)

statement 1 is correct

tr(A) = a+d = 0 , $\{\because a = -d\}$

statement 2 is false

6 Thee statement $p \to (q \to q)$ is equivalent to

(1) $p \longrightarrow (p \longleftrightarrow q)$

(2) $p \longrightarrow (p \longrightarrow q)$

(3) $p \longrightarrow (p \vee q)$

(4) $p \longrightarrow (p \wedge q)$

7. The value of cost $\left(\operatorname{cos}ec^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$ is

(1) $\frac{5}{17}$

(2) $\frac{6}{17}$

(3) $\frac{3}{17}$

(4) $\frac{4}{17}$

Ans

$$\cot\left(\tan^{-1}\frac{5}{3}+\tan^{-1}\frac{2}{3}\right)=\cot\left(\tan^{-1}\frac{3}{\sqrt{25-9}}+\tan^{-1}\frac{2}{3}\right)$$

$$=\cot\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right)=\cot\left(\tan^{-1}\frac{3/4+2/3}{1-6/12}\right)$$

$$\cot\left(\tan^{-1}\frac{17}{6}\right)=\cot\cot^{-1}\frac{6}{17}=\frac{6}{17}$$

8. The differential equation of the family of circles with fixed radius 5 units and centre on the line y = 2 is

(1) $(x-2)^2 y'^2 = 25-(y-2)^2$

(2) $(x-2)y'^2 = 25-(y-2)^2$

(3) $(y-2)y'^2 = 25-(y-2)^2$

(4) $(y-2)^2 y'^2 = 25-(y-2)^2$

Ans

C = (t, 2) r=5

$(x-t)^2 + (y-2)^2 = 25$

$$2(x-t)+2(y-2)\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{x-t}{2-y}$$

replace $(x-t)^2$

$$\Rightarrow y'^2=\frac{25-(y-2)^2}{(2-y)^2}$$

9. Let I = $\int_0^1 \frac{\sin x}{\sqrt{x}} dx$ and J= $\int_0^1 \frac{\cos x}{\sqrt{x}} dx.$

Then which one of the following is true?

(1) I > $\frac{2}{3}$ and J < 2

(2) I > $\frac{2}{3}$ and J > 2

(3) I < $\frac{2}{3}$ and J < 2

(4) I < $\frac{2}{3}$ and J > 2

Ans

$\forall o < x < 1,$ x > sinx

$$\frac{x}{\sqrt{x}} > \frac{\sin x}{\sqrt{x}} \Rightarrow \int_0^1 \sqrt{x} dx > \int_0^1 \frac{\sin x}{\sqrt{x}} \Rightarrow 2/3 > I \Rightarrow I < 2/3$$

$\forall 0 < x < 1 \quad \cos x < 1$

$$\Rightarrow \frac{\cos x}{\sqrt{x}} < \frac{1}{\sqrt{x}} \Rightarrow \int_0^1 \frac{\cos x}{\sqrt{x}} dx < \int_0^1 \frac{dx}{\sqrt{x}} \Rightarrow J < 2\sqrt{x}\Big|_0^1 \Rightarrow J > 2$$

10. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to

(1) $\frac{4}{3}$

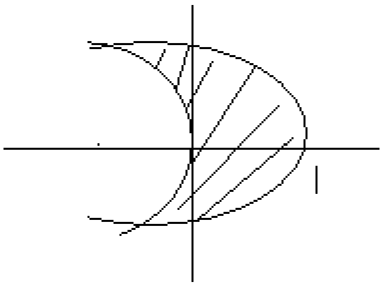
(2) $\frac{5}{3}$

(3) $\frac{1}{3}$

(4) $\frac{2}{3}$

Ans

$2y^2 = -x$ and $3y^2 = -(x-1) \Rightarrow x + 3y^2 = 1$



solve $2y^2 = -x$ and $x + 3y^2 = 1$

$$\Rightarrow x + \frac{3}{2}(-x) = 1 \Rightarrow \frac{-x}{2} = 1 \Rightarrow x = -2$$

$$Area = 2\int_0^1 \left[\left(1 - 3y^2\right) - \left(-2y^2\right)\right] dy$$

$$= 2\int_0^1 (1 - y^2)dy = 2\left[y - \frac{y^3}{3}\right]_0^1 = 2(1 - 1/3) = 4/3$$

11. The value of

$$\sqrt{2}\int \frac{\sin x dx}{\sin\left(x - \frac{\pi}{4}\right)} \text{ is}$$

(1) x - log | cos (x - $\frac{\pi}{4}$) | + c

(2) x + log | cos (x - $\frac{\pi}{4}$) | + c

(3) x - log | sin (x - $\frac{\pi}{4}$) | + c

(4) x + log | sin (x- $\frac{\pi}{4}$) | + c

Ans

$$\sqrt{2}\int \frac{\sin x dx}{\frac{\sin x}{\sqrt{2}} - \frac{\cos x}{\sqrt{2}}} = 2\int \frac{\sin x dx}{\sin x - \cos x}$$

$$= \int \frac{(\sin x - \cos x) + (\sin x + \cos x)}{\sin x - \cos x} dx$$

$$\int dx + \int \frac{\sin x + \cos x}{\sin x - \cos x} dx = x + \log|\sin x - \cos x| + C$$

$$x + \ln \frac{1}{\sqrt{2}} + \ln|\sin x - \cos x| + c'$$

$$= x + \ln|\sin(x - \pi/4)| + c'$$

12. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60^o. He moves away from the pole along the line BC to a point D such that CD = 7 m. From D the angle of elevation of the point A is 45^o. Then the height of the pole is

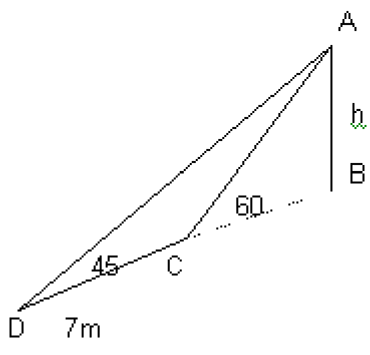
(1) $\frac{7\sqrt{3}}{2} \frac{1}{\sqrt{3}+1} m$

(2) $\frac{7\sqrt{3}}{2} \frac{1}{\sqrt{3}-1} m$

(3) $\frac{7\sqrt{3}}{2} \left(\sqrt{3}+1\right) m$

(4) $\frac{7\sqrt{3}}{2} \left(\sqrt{3}-1\right) m$

Ans



in ΔADB

h cot 45^o = 7 + BC ------(1)

in ΔABC

$h \cot 60^o = BD$ ----(2)

subtract 2 from 1,

$$h\cot 45^0 - h\cot 60^0 = 7 \Rightarrow h - \frac{h}{\sqrt{3}} = 7$$

$$h = \frac{7\sqrt{3}}{\sqrt{3}-1} = \frac{7\sqrt{3}\left(\sqrt{3}+1\right)}{2}$$

13. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x = 0$ have ?

(1) 5 (2) 7 (3) 1 (4) 3

Ans

$$f(x) = x^7 + 14x^5 + 16x^3 + 30x - 560$$

$$f'(x) = 7x^6 + 70x^4 + 48x^2 + 30$$

$$f'(x) > 0 \quad \forall x \in R$$

$$\lim_{x\to\infty} f(x) = \infty > 0,\ f(0) = -560 < 0$$

=> f(x) crosses x-axis only once

14.

$$Let\ \ f(x) = \begin{cases} (x-1)\sin\frac{1}{x-1} & if\ \ x \neq 1 \\ 0 & if\ \ x = 1 \end{cases}$$

Then which one of the following is true ?

(1) f is differentiable at x = 1 but not at x = 0
(2) f is neither differentiable at x = 0 nor at x = 1
(3) f is differentiable at x = 0 and x = 1
(4) f is differentiable at x = 0 but not at x = 1

Ans

$$f(x) = (x-1)\sin\left(\frac{1}{x-1}\right)$$

RHD at x=1

$$= \lim_{h\to 0} \frac{(1+h-1)\sin\left(\frac{1}{1+h-1}\right) - 0}{h}$$

$$= \lim_{h\to 0} \frac{h\sin\frac{1}{h}}{h} = \lim_{h\to 0}\sin\frac{1}{h}$$

= Oscillating between -1 & 1

=> RHD does not exists

Non diff. At x=1

At x=0, f(x) is continuous & diff. As both (x-1) and sin(1/x-1) are continuous & diff. At x=0

15. The first two terms of a geometric progression add up to 12. The sum if the third and the fourth terms is 48. If the terms of the geometric progression ate alternately positive and negative, then the first term is

(1) 4

(2) –4

(3) –12

(4) 12

Ans

Given b+br = 12 ---(1)

$br^2 + br^3 = 48$ r<0

$b(1+r) = 12$ & $br^2 (1+r) = 48$

Devide => $r^2 = 4$ => $r = \pm 2$

As r<0, we take r = -2

Replace r in (1) to get

b(1-2) = 12 => b = -12

16. It is given that the events A and B are such that P(A) = $\frac{1}{4}$, P(A | B) = $\frac{1}{2}$ and P(B | A) = $\frac{2}{3}$. Then P(B) is

(1) $\frac{1}{2}$

(2) $\frac{1}{6}$

(3) $\frac{1}{3}$

(4) $\frac{2}{3}$

Ans

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \;\&\; P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow \frac{P(A/B)}{P(B/A)} = \frac{P(A)}{P(B)} \Rightarrow P(B) = \frac{2/3 \times 1/4}{1/2} = \frac{1}{3}$$

17. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is

(1) $\frac{2}{5}$

(2) $\frac{3}{5}$

(3) 0

(4) 1

Ans

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$=\frac{3}{6}+\frac{4}{6}-\frac{1}{6}=\frac{6}{6}=1$$

18. Suppose the cubic $x^3 - px + q$ has three distinet real roots where $p > 0$ and $q > 0$. Then which one of the following holds ?

(1) The cubic has maxima at both $\sqrt{\frac{P}{3}}$ and $-\sqrt{\frac{P}{3}}$

(2) The cubic has minima at $\sqrt{\frac{P}{3}}$ and maxima at $-\sqrt{\frac{P}{3}}$

(3) The cubic has minima at $-\sqrt{\frac{P}{3}}$ and maxima at $\sqrt{\frac{P}{3}}$

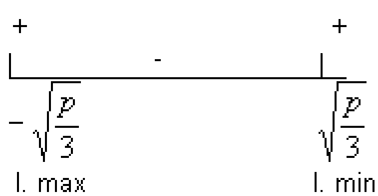
(4) The cubic has minima at both $\sqrt{\frac{P}{3}}$ and $-\sqrt{\frac{P}{3}}$

Ans

let $f(x) = x^3-px+q$

$f'(x) = 3x^2 -p$

$$=3\left(x-\sqrt{\frac{p}{3}}\right)\left(x+\sqrt{\frac{p}{3}}\right)$$



19. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent ?

(1) $7 \bullet {}^6C_4 \bullet {}^8C_4$

(2) $8 \bullet {}^6C_4 \bullet {}^7C_4$

(3) $6 \bullet 7 \bullet {}^8C_4$

(4) $6 \bullet 8 \bullet {}^{7}C_4$

Ans

MIIIPPI SSSS

Arrange MIIIPPI in

$\frac{7!}{4!\,2!}$ Between 7 letter there are 8 possibilities for 4

select 4 position out of 8 in ${}^{8}C_4$ ways

$$\Rightarrow ways = \frac{7!}{4!\,2!}\,{}^{8}C_4 = \frac{7 \times 6 \times 5}{2}\,{}^{8}C_4 = (7 \times 15)\,{}^{8}C_4$$

$$= 7 \times {}^{6}C_4 \times {}^{8}C_4$$

20. The perpendicular bisector of the line segment joining P(1, 4) and Q(k, 3) has y-intercept –4. Then a possible value of k is

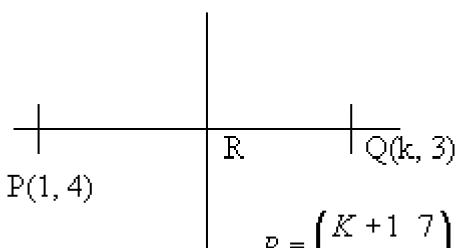
(1) -4

(2) 1

(3) 2

(4) –2

Ans



$$R \equiv \left(\frac{K+1}{2}, \frac{7}{2}\right)$$

equation of $\perp$ bisector

$$y-\frac{7}{2}=-\frac{(K-1)}{(3-4)}\left(x-\frac{K+1}{2}\right)$$

Put x=0 & y = -4

$$-4-\frac{7}{2}=-\frac{K-1}{-1}\left[\frac{-(K+1)}{2}\right]\Rightarrow\frac{-15}{2}=-\frac{(K-1)(K+1)}{2}$$

$$\Rightarrow K^2-1=15\Rightarrow K=\pm4$$

21. A parabola has the origin as its focus and the line x = 2 as the directrix. Then the vertex of the parabola is at

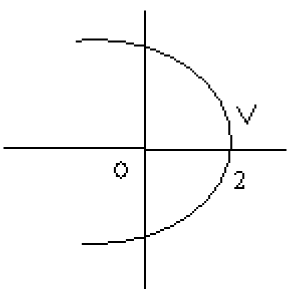
    (1) (2, 0)

    (2) (0, 2)

    (3) (1, 0)

    (4) (0, 1)

Ans



distance of vertex from directrix = distance from focus

=> V=(1, 0)

22. The point diametrically opposite to the point P(1, 0) on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is

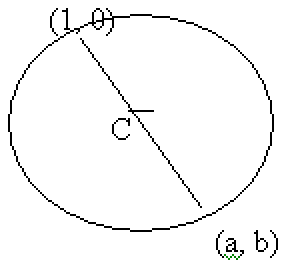
    (1) (3, 4)

    (2) (3, -4)

    (3) (-3, 4)

    (4) (-3, -4)

Ans



$C \equiv (-1,-2)$

$\frac{a+1}{2} = -1 \Rightarrow a = -3$

$\frac{b+0}{2} = -2 \Rightarrow b = -4$

$\Rightarrow (a,b) = (-3,-4)$

23. A focus of an ellipse is at the origin. The directrix is the line x = 4 and the eccentricity is $\frac{1}{2}$. Then the length of the semi-major axis is

(1) $\frac{5}{3}$ (2) $\frac{8}{3}$ (3) $\frac{2}{3}$ (4) $\frac{4}{3}$

Ans

$e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{1}{2}$

equation of ellipse

$\frac{\text{distance of P from focus}}{\text{distance of P from directrix}} = C$

$\Rightarrow \frac{\sqrt{x^2+y^2}}{|x-4|} = e \Rightarrow x^2 + y^2 = e^2(x-4)^2$

$\Rightarrow x^2 + y^2 = \frac{1}{4}(x-4)^2$

$\Rightarrow 3x^2 + 8x + 4y^2 = 16$

$$\Rightarrow 3\left[x^2 + 2\left(\frac{4}{3}\right)x + \frac{16}{9}\right] - \frac{16}{3} + 4y^2 = 16$$

$$\Rightarrow 3(x+4/3)^2 + 4y^2 = \frac{64}{3}$$

$$\Rightarrow \frac{(x+4/3)^2}{\frac{64}{9}} + \frac{y^2}{\frac{64}{12}} = 1$$

$$a^2 = \frac{64}{9} \Rightarrow a = \frac{8}{3}$$

24. The solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$

satisfying the condition y(1) = 1 is

(1) y = x ln x + x

(2) y = ln x + x

(3) y = x ln x x + x^2

(4) y = x $e^{(x-1)}$

Ans

$$\frac{dy}{dx} = 1 + \frac{y}{x} \Rightarrow \frac{dy}{dx} - \frac{y}{x} = 1$$ (linear DE)

IF= $e^{\int -\frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$

$$y\frac{1}{x} = \int 1\left(\frac{1}{x}\right)dx + C \Rightarrow \frac{y}{x}\ln|x| + C$$

$$y(1) = 1 \Rightarrow \frac{1}{1} = \ln|1| + C \Rightarrow C = 1$$

$$\Rightarrow \frac{y}{x} = \ln|x| + 1 \Rightarrow y = x\ln|x| + x$$

25. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that x = cy + bz, y = az + cz, and z = bx +ay. Then $a^2 + b^2 + c^2 + 2abc$ is equal to

(1) 1

(2) 2

(3) –1

(4) 0

Ans

x-cy-bz=0

cx-y+az=0

bx+ay-z=0

It is given that system of equations is consistent.

i.e. posses a solution.

It is given that not all x, y, z are zero.

=> Non trivial solution exist

=> D=0

$$\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

Evalute $1(-1-a^2)+c(-c-ab)-b(ac+b)=0$

$\Rightarrow -1-a^2-c^2-abc-abc-b^2 = 0$

$\Rightarrow a^2+b^2+c^2+2abc = -1$

26. Let A be a square matrix all of whose entries are integers. Then which one of the following is true ?

(1) If det A = ± 1, then A^{-1} need not exist

(2) If det A = ± 1, then A^{-1} exists but all its entries are not necessarily integers

(3) If det A $\neq \pm 1$, then A^{-1} exists and all its entries are non-integers.

(4) If det A = ± 1, then A^{-1} exists and all its entries are integers

Ans

For example, let

$\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} = A$

det(A) = 1

$A^{-1} = \begin{bmatrix} 2 & 5 \\ -1 & 3 \end{bmatrix}$

A^{-1} exists & all entries are integers.

27. The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$

have one root in common. The other roots of the first and second equations are integers in the ration 4: 3. Then the common root is

(1) 2 (2) 1 (3) 4 (4) 3

Ans

Let common root be α

$\alpha^2 - 6\alpha + a = 0$ &

$\alpha^2 - c\alpha + 6 = 0$

subtract,

$(c-6)\alpha = 6 - a$

$\Rightarrow \alpha = \frac{6-a}{c-6} - - - - - (1)$

other root of first equation = $\frac{a}{\alpha}$

other root of second equation = $6/\alpha$

Ration of other root = 4/3

$\Rightarrow a/6 = 4/3 \Rightarrow a = 8$

Replace a = 8 in first quadratic,

$\alpha^2 - 6\alpha + 8 = 0 \Rightarrow (\alpha-4)(\alpha-2) = 0 \Rightarrow \alpha = 4,2$

28. The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80. Then which one of the following gives possible values of a and b ?

(1) a = 3, b = 4

(2) a = 0, b = 7

(3) a = 5, b = 2

(4) a = 1, b = 6

Ans

$$\frac{a+b+8+5+10}{5} = 6 ----(1)$$

$$(a-6)^2 + (b-6)^2 + (8-6)^2 + (5-6)^2 + (10-6)^2 = 6.8 \times 5$$

--------(2)

simplify (1) to get:

23+a+b=30 => a+b = 7 -----(3)

simplify (2) to get

$(a-6)^2+(b-6)^2+4+1+16=34$

=> $(a-6)^2+(7-a-6)^2 = 13$

$(a^2-12a+36)+(a^2+1-2a) = 13$

$2a^2-14a+24 = 0$

$a^2-7a+12 = 0$ => (a-4)(a-3)=0 => b = 3, or 4

29. The vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ lies in the plane of the vectors $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{j} + \hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α *and* β?

(1) $\alpha = 1,\ \beta = 1$

(2) $\alpha = 2,\ \beta = 2$

(3) $\alpha = 1,\ \beta = 2$

(4) $\alpha = 2,\ \beta = 1$

Ans

$$\bar{a} = K(\hat{b} + \hat{c}) \Rightarrow K\left(\frac{\hat{i} + \hat{j}}{\sqrt{2}} + \frac{\hat{j} + \hat{k}}{\sqrt{2}}\right)$$

$$\bar{a} = \frac{K}{\sqrt{2}}(\hat{i} + 2\hat{j} + \hat{k})$$

$$\Rightarrow \alpha\hat{i} + 2\hat{j} + \beta\hat{k} = \frac{K}{\sqrt{2}}(\hat{i} + 2\hat{j} + \hat{k})$$

Comparing coeff. of $\hat{j}$

$$2 = \frac{K}{\sqrt{2}}(2) \Rightarrow K = \sqrt{2}$$

comparing coeff. of $\hat{i}$ & $\hat{k}$

$\alpha = 1, \beta = 1$

30. The non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are related by $\vec{a} = 8\vec{b}$ and $\vec{a} = -7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$ is

(1) π

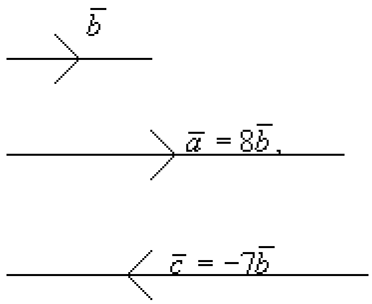
(2) 0

(3) $\frac{\pi}{4}$

(4) $\frac{\pi}{2}$

Ans

$\bar{a} = 8\bar{b}, \ \bar{c} = -7\bar{b}$



$\bar{a}$ & $\bar{c}$ are anti- parallel angle between them is π

31. The line passing through the points (5, 1, a) and (3, b, 1) crosses the yz-plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then

(1) a = 8, b = 2

(2) a = 2, b = 8

(3) a = 4, b = 6

(4) a = 6, b = 4

Ans

Equation of line is

$$l \equiv \frac{x-5}{2} = \frac{y-1}{1-b} = \frac{z-a}{a-1} = k$$

let coordinates of intersection of l with yz plane be [2k+5, (1-b)k+1, (a-1)k+a]

it lies on yz plane,

2k+5 = 0 => k = -5/2

Also (1-b)k+1=17/2 => 1-b = 15/2(-2/5) = -3 => b =4

Also (a-1)k+a =-13/2

A(1+k)-k = -13/2 => a(1-5/2) = -13/2-5/2 = -9

=> -3/2 a = -9 => a = 6

32. If the straight lines $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-2}{3}$ *and* $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$ intersect at a point, then the integer k is equal to

(1) -2

(2) –5

(3) 5

(4) 2

Ans

Let given be

$\bar{r} = \bar{a}_1 + K\bar{b}_1$ &

$\bar{r} = \bar{a}_2 + K\bar{b}_2$

lines are intersecting if $(\bar{a}_1 - \bar{a}_2).(\bar{b}_1 \times \bar{b}_2) = 0$

=> scaler triple product if

$(\bar{a}_1 - \bar{a}_2), \bar{b}_1 \,\&\, \bar{b}_2$ is 0

$\Rightarrow [\bar{a}_1 - \bar{a}_2 \quad \bar{b}_1 \quad \bar{b}_2] = 0$

$$\begin{vmatrix} 1 & 1 & -2 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix} = 0$$

$= 1(4-3k) - 1(2k-9) - 2(k^2-6) = 0$

$-2k^2 - 5k + 25 = 0 \Rightarrow +2k^2 + 5k - 25 = 0$

$2k^2 + 10k - 5k - 25 = 0 \Rightarrow (k+5)(2k-5) = 0$

$k = -5, k = 5/2$

33. The conjugate of a complex number is $\frac{1}{i-1}$. Then that complex number is

(1) $\frac{1}{i-1}$. (2) $\frac{-1}{i-1}$. (3) $\frac{1}{i+1}$. (4) $\frac{-1}{i+1}$.

Ans

$$\overline{Z} = \frac{1}{i-1}$$

$$Z = \frac{1}{\overline{i-1}} = \frac{1}{-i-1} = \frac{-1}{1+i}$$

34. Let R be the real line. Consider the following subsets of the plane R $\times$ R:

S = {(x, y) : y = x + 1 and 0 < x < 2}

T = {(x, y) : x – y is an integer}.

Which one of the following is true ?

(1) T is an equivalence relation on R but S is not

(2) Neither S nor T is an equivalence relations on R

(3) Both S and T are equivalence relation on R

(4) S is an equivalence relation on R but T is not

35. Let f : N $\rightarrow$ Y be a function defined as f(x) =4x + 3 where

Y = {y $\in$ N : y = 4x + 3 for some x $\in$ N}. Show that f is invertible and its inverse is

(1) $g(y) = \frac{y-3}{4}$

(2) $g(y) = \frac{3y+4}{3}$

(3) $g(y) = \frac{y+3}{4}$

(4) $g(y) = \frac{y+3}{4}$

Ans

To find inverse of f, replace y by x and x by y.

i.e.

x = 4f $^{-1}$(x)+3 => f $^{-1}$(x) = g(x) = x-3/4

Interms of y

g(y) = y-3/4